GEOMETRY

The Diocesan High School Mathematics Learning Outcomes/Standards contain three components; an introduction, an overview, a set of essential questions and enduring understandings, and standards that are closely aligned with the Massachusetts Frameworks.

Introduction:

The fundamental purpose of the Model Geometry course is to formalize and extend students' geometric experiences from the middle grades. This course is comprised of standards selected from the high school **conceptual categories**, which were written to encompass the scope of content and skills to be addressed throughout grades 9–12 rather than through any single course. Therefore, the complete standard is presented in the model course, with clarifying footnotes as needed to limit the scope of the standard and indicate what is appropriate for study in this particular course.

In this high school Model Geometry course, students explore more complex geometric situations and deepen their explanations of geometric relationships, presenting and hearing formal mathematical arguments. Important differences exist between this course and the historical approach taken in geometry classes. For example, transformations are emphasized in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found on page 92.

For the high school Model Geometry course, instructional time should focus on six critical areas: (1) establish criteria for congruence of triangles based on rigid motions; (2) establish criteria for similarity of triangles based on dilations and proportional reasoning; (3) informally develop explanations of circumference, area, and volume formulas; (4) apply the Pythagorean Theorem to the coordinate plan; (5) prove basic geometric theorems; and (6) extend work with probability.

(1) Students have prior experience with drawing triangles based on given measurements and performing rigid motions including translations, reflections, and rotations. They have used these to develop notions about what it means for two objects to be congruent. In this course, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats including deductive and inductive reasoning and proof by contradiction—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

(2) Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. Students derive the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on their work with quadratic equations done in Model Algebra I. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

(3) Students' experience with three-dimensional objects is extended to include informal explanations of circumference, area, and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

(4) Building on their work with the Pythagorean Theorem in eighth grade to find distances, students use the rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals, and slopes of parallel and perpendicular lines, which relates back to work done in the Model Algebra I course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

(5) Students prove basic theorems about circles, with particular attention to perpendicularity and inscribed angles, in order to see symmetry in circles and as an application of triangle congruence criteria. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an

equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations—which relates back to work done in the Model Algebra I course—to determine intersections between lines and circles or parabolas and between two circles.

(6) Building on probability concepts that began in the middle grades, students use the language of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Overview:

Number and Quantity

Quantities

- Reason quantitatively and use units to solve problems.

Geometry

Congruence

- Experiment with transformations in the plane.
- Understand congruence in terms of rigid motions.
- Prove geometric theorems.
- Make geometric constructions.

Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity in terms of similarity transformations.
- Prove theorems involving similarity.
- Define trigonometric ratios and solve problems involving right triangles.
- Apply trigonometry to general triangles.

Circles

- Understand and apply theorems about circles.
- Find arc lengths and area of sectors of circles.

Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a conic section.
- Use coordinates to prove simple geometric theorems algebraically.

Geometric Measurement and Dimension

- Explain volume formulas and use them to solve problems.
- Visualize relationships between two-dimensional and three-dimensional objects.

Modeling with Geometry

- Apply geometric concepts in modeling situations.

Statistics and Probability

Conditional Probability and the Rules of Probability

- Understand independence and conditional probability and use them to interpret data.
- Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Using Probability to Make Decisions

- Use probability to evaluate outcomes of decisions.

Essential Questions and Enduring Understandings:

Unit 1 – Essentials of Geometry

Essential Questions:

1. How can you represent a three-dimensional figure with two-dimensional drawing?
2. How do we represent points, lines, and planes?
3. What are the building blocks of geometry?
4. How can you describe the attributes of a segment or angle?

Enduring Understanding:

1. You can represent a three-dimensional object with a two-dimensional figure using special drawing techniques.
2. Geometry is a mathematical system built on accepted facts, basic terms, and definitions.
3. You can use number operations to find and compare lengths of segments, and the measure of angles.
4. Special angle pairs can help you identify geometric relationships and find missing angles.

Unit 2 – Reasoning and Proof

Essential Questions:

1. How can you make a conjecture and prove that it is true?
2. How do you construct a logical argument?

Enduring Understanding:

1. You can describe some mathematical relationships using a variety of *if-then* statements.
2. Given true statements, you can use deductive reasoning to make a valid or true conclusion.
3. You can use given information, definitions, properties, postulates, and previously proven theorems as reasons in a proof.

Unit 3 – Parallel and Perpendicular Lines/Planes

Essential Questions:

1. How do you prove that two lines are parallel?
2. What is the sum of the measures of the angles of a triangle?
3. What are the characteristics of intersecting, parallel, perpendicular, and skew lines?
4. How do you write an equation of a line in the coordinate plane?

Enduring Understandings:

1. Not all lines and not all planes intersect.
2. The special angle pairs formed by parallel lines and a transversal are congruent, supplementary, or both.
3. You can use certain angle pairs to decide whether two lines are parallel.
4. You can use the relationships of two lines to a third to decide whether the two lines are parallel or perpendicular to each other.
5. The sum of the angle measures of a triangle is always 180^{o}.
6. You can determine whether two lines are parallel or perpendicular by comparing their slopes.

Unit 4 – Congruent Triangles

Essential Questions:

1. What is congruence?
2. How can we prove one triangle is congruent to another?
3. How do you show that two triangles are congruent?
4. How can you tell whether a triangle is isosceles or equilateral?

Enduring Understandings:

1. Congruent figures have the same size and shape.
2. You can determine whether two figures are congruent by comparing their corresponding parts.
3. You can prove that two triangles are congruent without having to show that all corresponding parts are congruent.
4. If you know two triangles are congruent, then you know that every pair of their corresponding parts is also congruent.
5. The angles and sides of isosceles and equilateral triangles have special relationships.
6. In overlapping triangles you can sometimes use congruent corresponding parts of one pair of triangle to prove another pair is also congruent.

Unit 5 – Relationships Within Triangles

Essential Questions:

1. How do you use coordinate geometry to find relationships within triangles?
2. How do you find the third side of a triangle if you know the lengths of the other two sides?
3. How do you solve problems that involve measurements of triangles?

Enduring Understandings:

1. A point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment and vice versa.
2. The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
3. There is a relationship between the mid-segment of a triangle and the third side of the triangle.
4. The angles and sides of a triangle have special relationships that involve inequalities.

Unit 6 – Polygons

Essential Questions:

1. How can you find the sum of the measures of polygon angles?
2. How can you classify quadrilaterals?

Enduring Understandings:

1. The sum of the measures of the interior angles of an n-gon is $(n - 2)180$.
2. The sum of the measures of the exterior angles of any polygon is 360^{o}.
3. The special parallelograms (rhombus, rectangle, and square) have basic properties about their sides, angles, and diagonals that help identify them.
4. You can classify figures in the coordinate plane using the formulas for slope, distance, and midpoint.

Unit 7 – Similarity

Essential questions:

1. What is the difference between congruent figures and similar figures?
2. How do you use proportions to find side lengths in similar polygons?
3. How do you show two triangles are similar?
4. How do you calculate actual distance from a scale drawing?

Enduring Understandings:

1. If two polygons are similar, the ratio of any two corresponding lengths equals the scale factor.
2. There are multiple ways (for example: SSS, SAS, and AA) to prove that two triangles are similar.
3. You can use ratios and proportions to decide whether two polygons are similar and to find unknown side lengths of similar figures.
4. When you draw the altitude to the hypotenuse of a right triangle, you form three pairs of similar right triangles.

Unit 8 – Right Triangles and Trigonometry

Essential Questions:

1. What types of problems does trigonometry help us solve?
2. How do you find a side length or angle measure in a right triangle?
3. How do trigonometric ratios relate to similar right triangles?
4. How is trigonometry used to solve general triangles?

Enduring Understandings:

1. When given the lengths of any two sides of a right triangle, the length of the third side can be found by using the Pythagorean Theorem.
2. Certain combinations of side lengths and angle measures of triangles can determine ratios of other side lengths and angle measures.
3. You can use the angles of elevation and depression as the acute angles of right triangles formed by a horizontal distance and vertical height.
4. The relationships between angle measure and side lengths can be stated using trigonometric relations.

Unit 9 – Transformations

Essential Questions:

1. How can you change a figure's position without changing its size and shape?
2. How can you change a figure's size without changing its shape?
3. How can you represent a transformation in the coordinate plane?

Enduring Understanding:

1. You can change the position of a geometric figure so that the angle measures and the distance between any two points of a figure remain the same.

2. When you reflect a figure across a line, each point of the figure maps to another point the same distance from the line, but on the other side.
3. Rotations preserve distance, angle measures and orientation of figures.
4. You can use a scale factor to make a larger or smaller copy of a figure that is also similar to the original figure.

Unit 10 – Area and Perimeter

Essential Questions:

1. How do you find the perimeter and area of a polygon or find the circumference and area of a circle?
2. How do the perimeters and areas of similar polygons compare?
3. How is the ratio of the areas of two similar polygons related to the ratio of corresponding sides?

Enduring Understandings:

1. The area of the following figures can be calculated by using formulas: parallelogram, rectangle, square, triangle, trapezoid, rhombus, kite, regular polygon, and circle.
2. Ratios can be used to compare the perimeters and areas of similar figures.
3. You can use trigonometry to find the area of a regular polygon when you know the length of a side, radius, or apothem.
4. You can find the length of a part of a circle's circumference by relating it to an angle in the circle.

Unit 11 – Surface Area and Volume

Essential Questions:

1. How do you find surface area and volume of a solid?
2. How do the surface areas and volumes of similar solids compare?
3. How are the surface area and volume of a solid calculated?

Enduring Understandings:

1. Volume is the amount of space occupied by a solid figure in three-dimensions.
2. Surface area can be measured with the sum of the areas of the faces of the solid.
3. Ratios can be used to compare the areas and volumes of similar solids.
4. You can analyze a three-dimensional figure by using the relationships among its vertices, edges, and faces.

Unit 12 – Circles

Essential Questions:

1. How can relationships between angles and arcs in a circle be shown?
2. When lines intersect a circle or within a circle, how are the measures of resulting angles, arcs, and segments found?
3. How can you prove relationships between angles and arcs in a circle?
4. How do you find the equation of a circle in the coordinate plane?

Enduring Understanding:

1. Information about congruent parts of a circle or congruent circles can be used to find information about other parts of the circle or circles.
2. Angles formed by intersecting lines have a special relationship to the arcs the intersecting lines intercept.
3. Angles formed by intersecting lines have a special relationship to the related arcs formed when the lines intersect a circle.
4. You can write the equation of a circle on a coordinate plane given its center and radius.

Unit 13 – Probability

Essential Questions:

1. What is the difference between theoretical and experimental probability?
2. How can frequency tables be used to represent real life situations?
3. What does it mean for an event to be defined as random?

Enduring Understandings:

1. Probability is a measure of the likelihood that an event will occur.
2. You can use data organized in tables that show frequencies to find probabilities.
3. You can find the probability of compound events by using the probability of each part of the compound event.
4. You can use two-way frequency tables to organize data and identify sample spaces to approximate probabilities.
5. A random event has no bias or inclination toward any particular outcome.

Content Standards:

Conceptual Category: Number and Quantity	
Domain: Quantities	**N-Q**
Reason quantitatively and use units to solve problems.	
1)	Define appropriate quantities for the purpose of descriptive modeling.
2)	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
3)	MA.3.a. Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used to measure.

Conceptual Category: Geometry	
Domain: Congruence	G-CO
Experiment with transformations in the plane.	
1)	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2)	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3)	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4)	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5)	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Understand congruence in terms of rigid motions.	
6)	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7)	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8)	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
Prove geometric theorems.	
9)	Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
10)	Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
11)	Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*
	MA.11.a. Prove theorems about polygons. *Theorems include: measures of interior and exterior angles, properties of inscribed polygons.*
Make geometric constructions.	
12)	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
13)	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Conceptual Category: Geometry	
Domain: Similarity, Right Triangles, and Trigonometry	**G-SRT**
Understand similarity in terms of similarity transformations.	
1) Verify experimentally the properties of dilations given by a center and a scale factor:	
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	
2) Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
3) Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar.	
Prove theorems involving similarity.	
4) Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	
5) Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
Define trigonometric ratios and solve problems involving right triangles.	
6) Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	
7) Explain and use the relationship between the sine and cosine of complementary angles.	
8) Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	
Apply trigonometry to general triangles.	
9) (+) Derive the formula $A = \frac{1}{2}ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	
10) (+) Prove the Laws of Sines and Cosines and use them to solve problems.	
11) (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	

Conceptual Category: Geometry	
Domain: Circles	**G-C**
Understand and apply theorems about circles.	
1) Prove that all circles are similar.	
2) Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	
3) Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	
MA.3.a. Derive the formula for the relationship between the number of sides and sums of the interior and sums of the exterior angles of polygons and apply to the solutions of mathematical and contextual problems.	
4) (+) Construct a tangent line from a point outside a given circle to the circle.	
Find arc lengths and areas of sectors of circles.	
5) Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	

Conceptual Category: Geometry	
Domain: Expressing Geometric Properties with Equations	**G-GPE**
Translate between the geometric description and the equation for a conic section.	
1) Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	
2) Derive the equation of a parabola given a focus and directrix.	
Use coordinates to prove simple geometric theorems algebraically.	
3) Use coordinates to prove simple geometric theorems algebraically*. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*	

4)	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
5)	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
6)	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Conceptual Category: Geometry	
Domain: Geometric Measurement and Dimension	**G-GMD**
Explain volume formulas and use them to solve problems.	
1)	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
2)	(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
3)	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
Visualize relationships between two-dimensional and three-dimensional objects.	
4)	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Conceptual Category: Geometry	
Domain: Modeling with Geometry	**G-MG**
Apply geometric concepts in modeling situations.	
1)	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
2)	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3)	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
4)	MA.4. Use dimensional analysis for unit conversions to confirm that expressions and equations make sense.

Conceptual Category: Statistics and Probability	
Domain: Conditional Probability and the Rules of Probability	**S-CP**
Understand independence and conditional probability and use them to interpret data.	
1)	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2)	Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3)	Understand the conditional probability of *A* given *B* as $P(A \text{ and } B)/P(B)$, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
4)	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
5)	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
Use the rules of probability to compute probabilities of compound events in a uniform probability model.	
6)	Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.
7)	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
8)	(+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\|A) = P(B)P(A\|B)$, and interpret the answer in terms of the model.
9)	(+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Conceptual Category: Statistics and Probability	
Domain: Using Probability to Make Decisions	**S-MD**
Use probability to evaluate outcomes of decisions.	
1)	(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
2)	(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).