ALGEBRA I

The Diocesan High School Mathematics Learning Outcomes/Standards contain three components; an introduction, an overview, a set of essential questions and enduring understandings, and standards that are closely aligned with the Massachusetts Frameworks.

Introduction:

For the High School Algebra I course, instructional time should focus on four critical areas: (1) deepen and extend understanding of linear and exponential relationships; (2) contrast linear and exponential relationships with each other and engage in methods for analyzing, solving, and using quadratic functions; (3) extend the laws of exponents to square and cube roots; and (4) apply linear models to data that exhibit a linear trend.

(1) By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. In Algebra I, students analyze and explain the process of solving an equation and justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating among various forms of linear equations and inequalities, and use them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

(2) In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In Algebra I, students learn function notation and develop the concepts of domain and range. They focus on linear, quadratic, and exponential functions, including sequences, and also explore absolute value, step, and piecewise-defined functions; they interpret functions given graphically, numerically, symbolically, and verbally; translate between representations; and understand the limitations of various representations. Students build on and extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

(3) Students extend the laws of exponents to rational exponents involving square and cube roots and apply this new understanding of number; they strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions. Students become facile with algebraic manipulation, including rearranging and collecting terms, and factoring, identifying, and canceling common factors in rational expressions. Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

(4) Building upon their prior experiences with data, students explore a more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Overview:

Number and Quantity

The Real Number System

- Extend the properties of exponents to rational exponents.
- Use properties of rational and irrational numbers.

Quantities

- Reason quantitatively and use units to solve problems.

Algebra

Seeing Structure in Expressions

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

Arithmetic with Polynomials and Rational Expressions

- Perform arithmetic operations on polynomials.

Creating Equations

- Create equations that describe numbers or relationships.

Reasoning with Equations and Inequalities

- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.
- Solve systems of equations.
- Represent and solve equations and inequalities graphically.

Functions

Interpreting Functions

- Understand the concept of a function and use function notation.
- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.

Building Functions

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Linear, Quadratic, and Exponential Models

- Construct and compare linear, quadratic, and exponential models and solve problems.
- Interpret expressions for functions in terms of the situation they model.

Statistics and Probability

Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models.

Essential Questions and Enduring Understandings:

Unit 1 – Introduction to Algebra
Essential Questions:
1. What are the ways to represent quantities, patterns, and relationships?
2. How can mathematical relationships be represented in an equation or real-life situations?
3. How are properties modeled and used in Algebra?
4. How are algebraic properties applied in order to standardize computations?

Enduring Understandings:
1. Algebra uses symbols to represent quantities that are unknown or that vary.
2. Mathematical phrases and real-world relationships can be represented using symbols and operations.
3. Relationships that are always true for real numbers are called properties, which are rules used to rewrite and compare expressions.
4. Equations are used to represent the relationship between two quantities that have the same value.
5. Relationships that are always true for real numbers are called properties, which are rules used to rewrite and compare expressions.
6. Order of operations can be used to simplify variable expressions.

Unit 2 – Linear Equations
Essential Questions:
1. Why is it advantageous to use and solve linear equations algebraically?
2. How can I apply ratios, rates, percents, and problem solving properties to math and real life?
3. What are the ways in which a change in one variable can effect a change in another?
4. What methods can be used to transform an algebraic equation into an equivalent equation?

Enduring Understandings:
1. An equation is used to represent the relationship between two quantities that have the same value.
2. Equivalent equations are equations that have the same solution(s).
3. The solutions to any equation can be found using properties of equality and inverse operations to form a series of similar equivalent equations.
4. The properties of equality can be used repeatedly to isolate the variable.
5. To solve a literal equation uses the same methods as with any equation to isolate any particular variable.
6. Equations can describe, explain, and predict various aspects of the real world.

Unit 3 – Solving and Graphing Linear Inequalities
Essential Questions:
1. How are relationships between quantities that are not equal represented?
2. Can inequalities that appear to be different be equivalent?
3. What are the ways to solve inequalities?
4. How are graphs used to solve linear inequalities?

Enduring Understandings:
1. An inequality is a mathematical sentence that orders the value of two expressions.
2. A number line can be used to visually represent the values that satisfy an inequality.
3. The properties of inequality can be used to solve inequalities.
4. Solutions of a compound inequality can be found either by identifying where the solution sets of the distinct inequalities overlap or by combining the solution sets to form a larger solution set.

Unit 4 – Introduction to Function and Graphing
Essential Questions:
1. How can functions be represented and described?
2. Can functions describe real-world situations?
3. What does the slope of a line indicate about the line?
4. What information does the equation of a line give you?
5. How can you make predictions based on a scatter plot?
6. Why are some sets of ordered pairs functions, while others are not?

Enduring Understandings:
1. Graphs can be used to visually represent the relationship between two variable quantities as they both change.
2. The value of one variable may be uniquely determined by the value of another variable. Such relationships may be represented using tables, words, equations, sets of ordered pairs, and graphs.

3. The set of all solutions of an equation forms the equation's graph. A real-world graph should only show points that make sense in the given situation.
4. Many real-world functional relationships can be represented by equations. You can use an equation to find the solution of a given real-world problem.
5. The slope of a line is the ratio of the change in y (rise) to the change in x (run) and in real-world applications; slope is interpreted as the rate of change.
6. A function is a special type of relation in which each value in the domain is paired with exactly one value in the range.
7. A scatter plot shows whether there is a positive, negative or no correlation in the data.

Unit 5 – Systems of Equations and Inequalities

Essential Questions:

1. How can a system of equations or inequalities be solved?
2. Can systems of equations model real-world situations?

Enduring Understandings:

1. Systems of linear equations can be solved graphically or algebraically.
2. Linear systems can have one solution, no solution, or infinitely many solutions.
3. The solution of a system of inequalities is the region of the coordinate plane where the graphs of the individual inequalities overlap.
4. Systems of linear equations can be used to model real-world problems.

Unit 6 – Exponents and Exponential Functions

Essential Questions:

1. How can very large and very small numbers be represented using exponents?
2. How can expressions involving exponents be simplified?
3. What are the characteristics of exponential functions?

Enduring Understandings:

1. The properties of exponents can be extended to include zero, negative, and rational exponents.
2. Powers of 10 are an easy way to write and compare very large or very small numbers; this particular notation is termed Scientific Notation.
3. Properties of exponents can be used to simplify exponent expressions.
4. Exponential functions can model growth or decay of an initial amount.

Unit 7 – Polynomial Operations and Factoring

Essential Questions:

1. What is a polynomial?
2. How are the properties of real numbers related to polynomials?
3. How do you factor polynomials completely?
4. How do you solve polynomial equations in factored form?

Enduring Understandings:

1. A polynomial is a monomial or the sum of monomials.
2. Polynomials can be added or subtracted by combining like terms.
3. The properties of real numbers can be used to multiply a monomial by a polynomial or simplify the product of binomials.
4. To factor a polynomial means to write it as an equivalent product of two or more other polynomials.
5. To factor a polynomial completely means to write it as a product of its prime factors.

Unit 8 – Radical Expressions and Equations

Essential Questions:

1. How are radical expressions represented?
2. What are the characteristics of the square root function?

Enduring Understandings:

1. The square root of a number is a number that, multiplied by itself, will give that number.
2. Radical expressions can be simplified using multiplication and division properties of exponents.
3. Properties of real numbers can be used to perform operations with radical expressions.

Unit 9 – Quadratic Equations and Functions

Essential Questions:

1. What are the characteristics of quadratic functions?

2. What methods can be used to solve quadratic equations?
3. What is the discriminant and how does it help to determine the nature of the quadratic's solutions?
4. How can quadratic functions be used to model real-world situations?

Enduring Understandings:

1. A quadratic function is a type of nonlinear function that models certain situations where the rate of change is not constant.
2. The graph of a quadratic function is a symmetric curve with a highest or lowest point corresponding to a maximum or minimum value.
3. Quadratic equations can be solved by a variety of methods, including graphing, finding square roots, factoring by using the Zero-Product Property, and the Quadratic Formula.
4. The discriminant of a quadratic function tells the nature of the solutions of the function.

Unit 10 – Data Analysis and Probability

Essential Questions:

1. How can collecting and analyzing data help to make decisions or predictions?
2. How can one make and interpret different representations of data?
3. How is probability related to real-world events?

Enduring Understandings:

1. Standard measures that describe data from real-world situations can help you make estimates or decisions about the situation, or predictions about future occurrences.
2. Categorical and quantitative data are displayed in different types of graphs.
3. Dot plots and histograms can help you see the distribution of quantitative data.
4. Experimental probability is observed from an experiment, while theoretical probability is a calculated prediction.

Content Standards:

Conceptual Category: Number and Quantity	
Domain: The Real Number System	***N-RN***
Extend the properties of exponents to rational exponents.	
1) Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*	
2) Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
Use properties of rational and irrational numbers.	
1) Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	

Conceptual Category: Number and Quantity	
Domain: Quantities	***N-Q***
Reason quantitatively and use units to solve problems.	
1) Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	
2) Define appropriate quantities for the purpose of descriptive modeling.	
3) Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	
4) MA.3.a. Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used to measure.	

Conceptual Category: Algebra	
Domain: Seeing Structure in Expressions	**A-SSE**
Interpret the structure of expressions.	
1) Interpret expressions that represent a quantity in terms of its context.	
a) Interpret parts of an expression, such as terms, factors, and coefficients.	
b) Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1 + r)^n$ *as the product of P and a factor not depending on P.*	
2) Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	
Write expressions in equivalent forms to solve problems.	
3) Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	
a) Factor a quadratic expression to reveal the zeros of the function it defines.	
b) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	
c) Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	

Conceptual Category: Algebra	
Domain: Arithmetic with Polynomials and Rational Expressions	**A-APR**
Perform arithmetic operations on polynomials.	
1) Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	

Conceptual Category: Algebra	
Domain: Creating Equations	**A-CED**
Create equations that describe numbers or relationships.	
1)	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2)	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3)	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4)	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Conceptual Category: Algebra		
Domain: Reasoning with Equations and Inequalities		**A-REI**
Understand solving equations as a process of reasoning and explain the reasoning.		
1)	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
Solve equations and inequalities in one variable.		
2)	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	
3)	MA.3.a. Solve linear equations and inequalities in one variable involving absolute value.	
4)	Solve quadratic equations in one variable.	
	a)	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
	b)	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
	c)	MA.4.c. Demonstrate an understanding of the equivalence of factoring, completing the square, or using the quadratic formula to solve quadratic equations.
Solve systems of equations.		
5)	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	
6)	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	
7)	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = –3x$ *and the circle* $x^2 + y^2 = 3$.	
Represent and solve equations and inequalities graphically.		
8)	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	
9)	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	
10)	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	

Conceptual Category: Functions	
Domain: Interpreting Functions	F-IF
Understand the concept of a function and use function notation.	
1)	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2)	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

3)	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n + 1) = f(n) + f(n − 1) for n ≥ 1.*
Interpret functions that arise in applications in terms of the context.	
4)	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5)	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
6)	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
Analyze functions using different representations.	
7)	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
a)	Graph linear and quadratic functions and show intercepts, maxima, and minima.
b)	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c)	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8)	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
a)	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
b)	Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, *and* $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*
c)	MA.8.c. Translate among different representations of functions and relations: graphs, equations, point sets, and tables.
9)	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
10)	MA.10. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.

Conceptual Category: Functions

Domain: Building Functions	**F-BF**
Build a function that models a relationship between two quantities.	
1)	Write a function that describes a relationship between two quantities.
a)	Determine an explicit expression, a recursive process, or steps for calculation from a context.
b)	Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
2)	Write arithmetic and geometric sequences both recursively and with an explicit formula,[1] use them to model situations, and translate between the two forms.
Build new functions from existing functions.	
3)	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4)	Find inverse functions.
a)	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x + 1)/(x - 1)$ *for* $x \neq 1$.

Conceptual Category: Functions	
Domain: Linear, Quadratic, and Exponential Models	**F-LE**
Construct and compare linear, quadratic, and exponential models and solve problems.	
1) Distinguish between situations that can be modeled with linear functions and with exponential functions.	
a) Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.	
b) Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	
c) Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	
2) Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
3) Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	
Interpret expressions for functions in terms of the situation they model.	
4) Interpret the parameters in a linear or exponential function in terms of a context.	

Conceptual Category: Statistics and Probability	
Domain: Interpreting Categorical and Quantitative Data	**S-ID**
Summarize, represent, and interpret data on a single count or measurement variable.	
1) Represent data with plots on the real number line (dot plots, histograms, and box plots).	
2) Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	
3) Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	
4) Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	
Summarize, represent, and interpret data on two categorical and quantitative variables.	
5) Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	
6) Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	
a) Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*	
b) Informally assess the fit of a function by plotting and analyzing residuals.	
c) Fit a linear function for a scatter plot that suggests a linear association.	
Interpret linear models.	
7) Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	
8) Compute (using technology) and interpret the correlation coefficient of a linear fit.	
9) Distinguish between correlation and causation.	